

MATH 120 – Sample MIDTERM #2
Fall 2012

Name:

ID#:

Instructions:

- You have 50 minutes to complete all questions.
- This exam is closed book, no calculator.
- Marks will only be awarded for demonstrated work; the value of each question is indicated in square brackets [].
- The maximum score is ??.
- Cheating is lame and will have some unpleasant consequences.

1. T is a mapping defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + y \\ x \end{bmatrix}$$

- (a) [2] What is the domain and codomain of T ?
- (b) [3] Is T linear?
- (c) [3] Find the standard matrix of T if possible.
- (d) [2] Use the standard matrix to find

$$T\left(\begin{bmatrix} 5 \\ 4 \end{bmatrix}\right)$$

2. Let

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 4 \\ 3 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 2 & 4 & 0 \\ 3 & -1 & 0 & 0 \end{bmatrix},$$

Perform the operation or explain why it is not defined.

(a) $[2] A + B$.

(b) $[3] AB$.

3. (a) [6] Write parametric and normal equations of the plane that passes through the origin and contains the line L given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- (b) [5] Determine whether the line L given by

$$\frac{x}{2} = \frac{y-2}{3} = \frac{z+1}{1}$$

is parallel to the plane P given by the equation $2x + 4y - z = 0$.

If they are parallel, find the distance between them, if they are not parallel find the point of intersection.

4. [4] Find all the values of the parameter a such that the following linear system (i) is inconsistent; (ii) has infinitely many solutions; (iii) has exactly one solution.

$$\begin{aligned}2x - 7y + 3z &= 0 \\x - 5y + 4z &= -3 \\-2x + y + az &= 0\end{aligned}$$

5. (a) i. [2] Give the definition of a linear mapping (transformation) corresponding to a matrix $A \in \mathbb{R}^{m \times n}$.
- ii. [2] True or False? (include a short explanation) A system is inconsistent if the number of equations exceeds the number of unknowns (variables) of the system.
- iii. [2] True or False? (include a short explanation) The number of pivots in the REF of a matrix A gives the number of linear independent column vectors in A .